MA 125 6D, CALCULUS I

Test 3, November 4, 2015

Name (Print last name first): ....

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 7 points each.

1. Use a Riemann sum with $n = 3$ terms and the right endpoint rule to approximate $\int_1^2 x^3\,dx$. (You don't need to multiply or add the terms.)

① $\Delta x = \frac{b-a}{n} = \frac{2-1}{3} = \frac{1}{3}$

② $x_1 = a + \Delta x = 1 + \frac{1}{3} = \frac{4}{3}$

③ $x_2 = a + 2\Delta x = 1 + \frac{2}{3} = \frac{5}{3}$

$x_3 = b = 2$

$\Rightarrow \int_1^2 x^3\,dx = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$

$= \frac{4}{3}\cdot\frac{1}{3} + \frac{5}{3}\cdot\frac{1}{3} + 2\cdot\frac{1}{3} = \frac{4+5+6}{9} = \frac{15}{9} = \boxed{\frac{5}{3}}$

2. Evaluate $\int 3x^2(x^3+1)\,dx$

$u = x^3 + 1 \Rightarrow du = 3x^2\,dx$

$\Rightarrow \int 3x^2(x^3+1)\,dx = \int u\, 3x^2\,dx = \int u\,du$

$= \frac{1}{2}u^2 + C = \frac{1}{2}(x^3+1)^2 + C$

3. Find the average value of the function $f(x) = \sec^2(x)$ on the interval $[0, \cancel{\pi}]$. $\frac{\pi}{4}$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{\frac{\pi}{4}-0}\int_0^{\pi} \sec^2 x\,dx$$

$$= \frac{4}{\pi}\tan x\Big]_0^{\cancel{\pi}\,\frac{\pi}{4}} = \frac{4}{\pi}\,\cancel{\pi}\left(\tan\frac{\pi}{4} - \tan 0\right)$$

$$= \frac{4}{\pi}.$$

4. Evaluate $\int \frac{x^2+x+1}{\sqrt{x}}\,dx$

$$\cancel{u = x^2+x+1 \Rightarrow du = 2x+}$$

$$\int \frac{x^2+x+1}{\sqrt{x}}\,dx = \int \frac{x^2}{\sqrt{x}} + \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}\,dx$$

$$= \int x^{\frac{3}{2}} + x^{\frac{1}{2}} + x^{-\frac{1}{2}}\,dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C.$$

5. Evaluate $\int x^2\cos(x^3)\,dx$

$$u = \cancel{\cos}\,x^3 \Rightarrow dx = 3x^2\,dx$$

$$\Rightarrow \int x^2\cos(x^3)\,dx = \int \cos u\;x^2\,dx = \int \cos u\;\frac{1}{3}\,du$$

$$= \frac{1}{3}\sin u + C = \frac{1}{3}\sin x^3 + C$$

6. Evaluate $\int \sqrt{\sin(x)} \quad \cos(x)\,dx$.

$$u = \sin x \Rightarrow du = \cos x\,dx$$

$$\Rightarrow \int \sqrt{\sin x}\,\cos x\,dx = \int \sqrt{u}\,\cos x\,dx$$

$$= \int \sqrt{u}\,du = \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}\sin^{\frac{3}{2}} x + C$$

7. Evaluate $\int_{-2}^{2} \frac{\sin(x)}{x^2+1}\,dx$

$$f(x) = \frac{\sin x}{x^2+1} \Rightarrow f(-x) = \frac{\sin(-x)}{(-x)^2+1} = \frac{-\sin x}{x^2+1}$$

$$= -f(x) \Rightarrow f(x) : \text{odd function}$$

$$\Rightarrow \int_{-2}^{2} \frac{\sin x}{x^2+1}\,dx = 0$$

8. Use the Fundamental theorem of calculus to write an expresstion for the antiderivative of the function $g(x) = \cos(x^2)$.

$$G(x) = \int_a^x \cos(t^2)\,dt$$

PART II

All problems in Part II are 11 points each.

1. Evaluate $\int_0^1 (2x+1)\sqrt[5]{x^2+x}\,dx$

$$u = x^2 + x \Rightarrow du = (2x+1)\,dx$$

$$\Rightarrow \int (2x+1)\sqrt[5]{x^2+x}\,dx = \int u^{\frac{1}{5}}(2x+1)\,dx = \int u^{\frac{1}{5}}\,du$$

$$= \frac{5}{6}u^{\frac{6}{5}} + C = \frac{5}{6}(x^2+x)^{\frac{6}{5}} + C$$

$$\Rightarrow \int_0^1 (2x+1)\sqrt[5]{x^2+x}\,dx = \frac{5}{6}(x^2+x)^{\frac{6}{5}}\Big]_0^1 = \frac{5}{6}2^{\frac{6}{5}}.$$

2. Evaluate $\displaystyle\int \frac{x^2}{(x-1)^{50}}\,dx$

$$u = x-1 \Rightarrow du = dx \Rightarrow x^2 = (u+1)^2 = u^2+2u+1$$

$$\Rightarrow \int \frac{x^2}{(x-1)^{50}}\,dx = \int \frac{u^2+2u+1}{u^{50}}\,du$$

$$= \int u^{-48} + 2u^{-49} + u^{-50}\,du$$

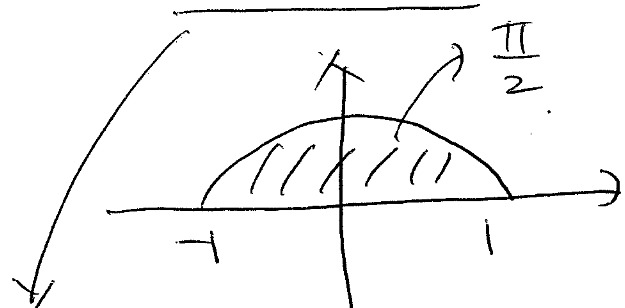
$$= -\frac{1}{47}u^{-47} - \frac{2}{48}u^{-48} - \frac{1}{49}u^{-49} + C$$

$$= -\frac{1}{47}(x-1)^{-47} - \frac{1}{24}(x-1)^{-48} - \frac{1}{49}(x-1)^{-49} + C$$

3. Evaluate $\int_{-1}^{1} \sqrt{1-x^2}\,dx$. (Hint: consider the graph.)

$y=\sqrt{1-x^2} \Rightarrow y^2 = 1-x^2 \Rightarrow \underline{x^2+y^2=1}$

$\Rightarrow \int_{-1}^{1} \sqrt{1-x^2}\,dx = \pi/2.$



Circle with radius 1.

4. Suppose the graph of a function $y = f(x)$ is shown in the plot below.
(a) Find the value of integral:

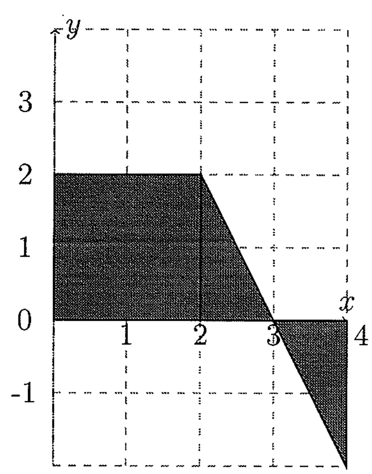
$$\int_0^4 f(x)dx \quad = 2\times 2 + 2\times 1\times \tfrac{1}{2} + 2\times 1\times \tfrac{1}{2}$$
$$= 4 + 1 + 1 = 6.$$

(b) Let $g(x) = \int_0^x f(t)dt$. Is $g(x)$ increasing or decreasing on $(3, 4)$?[As always you must explain your answer!]

$g'(x) = f(x)$

$\Rightarrow$ on $\boxed{(3,4)}$

The area of a Triangle is $\frac{1}{2}$ × base × height



$f(x)$ is negative.

$\Rightarrow g'(x) < 0$

$\Rightarrow g(x)$ decreasing.